

# Alpha-Cluster Model, Charge Symmetry of Nuclear Force and Single Particle Bound State Potential in Symmetrical Nuclei

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A phenomenological  $\alpha$ -cluster model based on the charge symmetry of nuclear force allows one to estimate the last proton position radius (LPPR) in a symmetrical nucleus. The values of LPPR obtained for the symmetrical nuclei with  $5 \leq Z \leq 45$  are used in a long standing problem of determination of the Woods-Saxon single particle bound state potential parameters. With respect to the charge symmetry of nuclear force a requirement of equality of the nuclear potentials for the last neutron and the last proton in a symmetrical nucleus is added to the standard well-depth procedure in solving the Shrödinger equation for the nucleon bound states, which makes the Coulomb radius the crucial parameter to determine the others and the value of the last proton rms radius. The Coulomb radii have been obtained with using LPPR. Analysis of the last proton rms radii, the Coulomb radii, the nuclear potential radii obtained at the calculations in comparison with LPPRs, the Coulomb radii from the  $\alpha$ -cluster model together with the experimental radii shows that for the nuclei with  $Z \geq 15$  it is inappropriate to represent a single particle bound state by the Woods-Saxon potential. For the nuclei with  $5 \leq Z \leq 14$  the error of the spectroscopic factor obtained with standard parameters in DWBA analysis of pure peripheral one nucleon transfer reactions is estimated. It is shown that for some nuclei using the standard parameters brings an error more than 20%.

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## I. INTRODUCTION

It is well known that nuclear properties are described in the framework of three different representations of nucleon arrangements [1], the mean field representation, the liquid drop model and the  $\alpha$ -cluster model. In analysis of one nucleon transfer reactions  $A(a, b)B$  where  $A = B + n/p$  and  $b = a + n/p$ , one has to use the mean field representation where neutrons and protons move independently in the mean field. The nuclear mean field is described by the Woods-Saxon (W-S) potential due to the single particle model [2] with parameters of radius  $r_0$ , diffuseness  $a_0$  and depth  $V_0$  depicting experimental charge distribution. The standard Distorted Wave Born Approximation (DWBA) approach [3, 4], used in the analysis of differential cross sections of one nucleon transfer reactions, suggests that the nucleon wave function with a factor  $S^{1/2}$  where  $S$  is the spectroscopic factor (SF), is equal to the overlap function of the nuclei  $A$  and  $B$ .  $S$  is obtained by dividing the experimental cross section of the reaction at the main scattering peak by the calculated one. The experimental binding energy of the nucleon (neutron/proton)  $\varepsilon_{n/p}$  in a nucleus  $A$  is used as the eigenvalue in the Shrödinger equation for the bound state.

The determination of the single particle bound state potential parameters is a long standing problem [5–15]. As the main criteria for selecting proper parameters the experimental nuclear radius and charge distribution are usually used [2, 5–9] with strong needs of additional

model parameters like sizes of internal shells and the binding energies of the nucleons of the shells, which do not make the obtained values valuable. During a few decades in the DWBA analysis so called standard parameters have been used, these are around the values  $r_0 = 1.25$  fm,  $a_0 = 0.65$  fm, which provide close values of the neutron and proton depths  $V_0 \approx 50$  MeV in so called well depth procedure adjusting the depth to the values  $\varepsilon_{n/p}$  [7–11]. Some articles, for example [10, 11], are devoted to finding so called global parameters of the W-S potential to describe single particle bound states of the nuclei united on some signs like closeness of the numbers of neutrons and protons to the magic ones. The parameters turned out to be close to the standard ones and they provide description of single particle spectra within an accuracy of 1-2 MeV [11]. In this connection the task of finding the realistic values of the parameters which are supposed to be unique for every nucleus as well as an estimation of the error in obtaining SF by means DWBA with standard parameters is very actual.

In [12] the task of finding the proper values of the parameters was redefined for the task of finding proper value of asymptotic coefficient (AC)  $b_{n/p}$  of the bound state wave function on the basis of the approach [16], where the asymptotic normalization coefficients of the bound state (ANC)  $C_{n/p}^2$  was introduced in the analysis. According to the approach the cross section of pure peripheral reaction at the main scattering peak of the angular distribution contains an asymptotic part of the nucleon's wave function which for the last neutron and the last proton are [12, 16]

$$\varphi_n(r)_{r \rightarrow \infty} = b_n i^l k h_{lj}^l(ikr) \quad (1)$$

$$\varphi_p(r)_{r \rightarrow \infty} = b_p W_{-\eta, lj}(2kr)/r, \quad (2)$$

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where  $h_l^j(ikr)$  is the spherical Hankel function of the first kind,  $W_{-\eta,lj}(2kr)$  is the Whittaker function,  $\eta$  is the Coulomb parameter for the bound state,  $k = \sqrt{2\mu\varepsilon_{n/p}}$ ,  $\mu$  is the reduced mass for the bound system  $A = B + n/p$ . AC  $b_{n/p}$  is determined by the geometrical parameters of the potential. Then ANC  $C_{n/p}^2$  is related with SF and AC as follows [12, 16]

$$C_{n/p}^2 = S b_{n/p}^2. \quad (3)$$

This approach allows one to obtain ANCs from analysis of experimental cross sections of pure peripheral reactions like  $A(d, t)B$  and  $B(^3\text{He}, d)A$  [17–19]. The value ANC obtained from the reactions does not depend of the potential parameters used in calculation, because in that case  $S$  is in inverse proportion to the  $b_{n/p}^2$ . In [12] it was proposed to obtain AC and corresponding parameters from analysis of experimental cross sections of direct reactions with using a known value ANC. But later it was shown [17] that peripheral reactions are not good for the task. In case of reactions like  $(p, d)$  and  $(d, n)$  there is a strong dependence of the calculated cross sections on the optical potentials used for input and outgoing channels, which does not allow one to solve the task either.

In [13] a method was developed where the parameters are obtained under the condition of the exact equivalence of the neutron and proton potentials for symmetrical and mirror nuclei (EPN condition) with respect to the charge symmetry of nuclear force. Together with the well- depth standard procedure the EPN condition makes the parameters dependent on each other, so the Coulomb radius  $r_C^{DWBA}$  becomes the main critical parameter to determine the value of root mean square (rms) radius calculated for the last proton  $\langle r_p^2 \rangle^{1/2}$  and AC. Varying one of the parameters, for example diffuseness  $a_0$ , at  $r_C^{DWBA}$  fixed brings some particular values of  $r_0$  and  $V_0$  so that  $\langle r_p^2 \rangle^{1/2}$  changes within 1% and  $b_{n/p}$  changes within a few per cent [14]. Moreover, the ratio of squared neutron and proton ACs for mirror and symmetrical bound states  $b_n^2/b_p^2$  is quite a stable value (with variation within 2%) at a wide variation of the Coulomb radius [15]. Some explanation why the ratio is to be stable is offered in [20]. The value  $b_n^2/b_p^2$  can be used to predict the ratio  $C_n^2/C_p^2$  for mirror and symmetrical bound states, because the SFs in the framework of the shell model [21] are equal. In [14] to obtain some criterion to determine the Coulomb radius for the nuclei of 1p shell some assumptions were made on the size of 1s shell, which diminished the validity of the obtained values.

In the present work like in [13–15] the EPN condition is used to obtain the potential parameters and AC. Besides  $\varepsilon_{n/p}$ , which is calculated as the difference between the binding energies of the nuclei  $A$  and  $B$ , another macroscopic quantity is used. That is the last proton position radius (LPPR)  $R_p$  in the center of mass system (cms) of the symmetric nucleus  $A = B + n/p$ . The value of LPPR is estimated from analysis of the nuclear binding energies in the framework of an  $\alpha$ -cluster model based on  $np$ -pair

interactions with using charge symmetry of nuclear force acting between the nucleons belonging to the  $\alpha$ -cluster matter [22–27]. One of the main aims of developing the model was finding proofs of validity of the EPN condition for single particle bound states in symmetrical and mirror nuclei.

The Coulomb radius of the last proton interaction with the residual nucleus  $R_{Cp}^{DWBA} = r_C^{DWBA}(A-1)^{1/3}$  is defined from the formula used for the Coulomb potential in the standard DWBA (see the manual to DWUCK program [4])

$$E_p^C(r) = \frac{(Z-1)e^2}{2R_{Cp}^{DWBA}} \left( 3 - \left( \frac{r}{R_{Cp}^{DWBA}} \right)^2 \right) \quad (4)$$

at  $r < R_{Cp}^{DWBA}$  and

$$E_p^C(r) = \frac{(Z-1)e^2}{r} \quad (5)$$

at  $r \geq R_{Cp}^{DWBA}$  where the Coulomb energy  $E_p^C$  of the last proton

$$E_p^C(r) = \Delta E_{np}, \quad (6)$$

and  $\Delta E_{np} = \varepsilon_n - \varepsilon_p$ . Eq. (6) is the consequence of the EPN condition. Taking into account the observed fact that the Schrödinger equation at the EPN condition provides the value of rms radius  $\langle r_p^2 \rangle^{1/2}$  less than  $R_{Cp}^{DWBA}$  Eq. (4) is used to find the value  $R_{Cp}^{DWBA}$  (or the parameter  $r_C^{DWBA}$ ) at  $r = R_p$ .

The article consists of 4 sections. The second section explains how the values LPPR  $R_p$  are obtained in the framework of the  $\alpha$ -cluster model for the symmetrical nuclei with  $5 \leq Z \leq 45$ . In the third section the parameters obtained by means of the EPN method are given. In the section 4 there are discussions of the results.

## II. RLPP OBTAINED IN ALPHA-CLUSTER MODEL

The  $\alpha$ -cluster model [22–27] is based on the charge symmetry of nuclear force, so the Coulomb energy of the last proton in a symmetrical nucleus is to be equal to  $\Delta E_{np}$  (6). According to representation of an  $\alpha$ -cluster liquid drop the nuclear matter is incompressible, so the Coulomb energy of a nucleus equals the energy of the  $np$ -pairs consisting the  $\alpha$ -clusters. Then for the nucleus with even  $Z$ , with the number of  $\alpha$ -clusters  $N_\alpha = Z/2$ , and for the nucleus with odd  $Z_1 = Z + 1$ , the number of  $\alpha$ -clusters  $N_\alpha + 0.5$ , the empirical value of the Coulomb energy is supposed to be estimated as follows [25]

$$E^C = \sum_{1}^{N_\alpha} (\Delta E_{np1} + \Delta E_{np})$$

$$E_1^C = E^C + \Delta E_{np1}, \quad (7)$$

where  $\Delta E_{np1}$  and  $\Delta E_{np}$  are the differences of one  $np$ -pair neutron and the proton binding energies, odd  $np$ -pairs have index 1. From the analysis of the experimental binding energies and the Coulomb energies (7) of the lightest nuclei with  $Z, Z_1 \leq 8$  some important values were obtained [24, 25]. These are the binding energy and the Coulomb energy of one  $\alpha$ -cluster (absolute values are given)  $\varepsilon_\alpha = \varepsilon_{4\text{He}}$  ( $\varepsilon_{4\text{He}} = 28.296$  MeV [28]) and  $\varepsilon_\alpha^C = 0.764$  MeV, the binding energy and the Coulomb energy of a cluster-cluster interaction  $\varepsilon_{\alpha\alpha} = 2.425$  MeV and  $\varepsilon_{\alpha\alpha}^C = 1.925$  MeV and the Coulomb energy of the interaction of the single  $np$ -pair in odd  $Z_1$  nucleus with the  $\alpha$ -cluster of its close vicinity  $\varepsilon_{np\alpha}^C = 1.001$  MeV. Also a simple formula to describe the binding energies of symmetrical nuclei with  $Z, Z_1 \leq 29$  was found [22]  $E = N_\alpha \varepsilon_\alpha + 3(N_\alpha - 2)\varepsilon_{\alpha\alpha}$  for the nuclei with even  $Z$  and  $E_1 = E + 14$  MeV for the nuclei with odd  $Z_1$ , which means that one added  $\alpha$ -cluster brings three new links with the closest ones. The formula means that the long range Coulomb energy must be compensated by the surface tension energy, which allows one to find a formula to calculate the latter. Thus a successful formula to calculate binding energy for stable and beta-stable nuclei as well as for the nuclei around the stable valley has been found [25]. In the formula the binding energy of  $\alpha$ -clusters is calculated separately from the energy of excess neutron pairs ( $nn$ -pairs). The accuracy of the calculation is comparable with the Weizsäcker formula [29], but unlike this well known formula the parameters used in the model are not fitting ones, they had been found from analysis of the binding energies of reduced amount of symmetrical nuclei. Another important finding proving the  $\alpha$ -cluster representation is that the nuclear radius for a stable and a beta-stable nucleus is defined by the number of the  $\alpha$ -clusters rather than by the total number of the nucleons. The simplest formula for nuclear radius growing with  $Z, Z_1$  is as follows [24, 25]

$$R = R_\alpha N_\alpha^{1/3}; R_1 = R_\alpha (N_\alpha + 0.5)^{1/3}, \quad (8)$$

where  $R_\alpha = R_{4\text{He}}$  ( $R_{4\text{He}} = 1.71$  fm [30]) for the nuclei with  $Z=2$  and  $5 \leq Z, Z_1 \leq 10$ . For the nuclei with  $Z, Z_1 > 24$   $R_\alpha = 1.595$  fm. To widen the number of nuclei to be described to the ones with  $N \geq Z, Z_1$  the model was developed to the representation of nucleus as a core (liquid  $\alpha$ -cluster drop with dissolved  $nn$ -pairs in it) and a molecule of a few  $\alpha$ -clusters on its surface [26, 27]. The notion of a nuclear molecule on the surface of a core was developed in [31]. The number of the molecule's  $\alpha$ -clusters is obtained from analysis of the nuclear binding energy. So for the nuclei with  $10 < Z, Z_1 \leq 24$  in (8) the radius  $R, R_1$  is defined by the sum of the volumes of the surface molecule, presumably the nucleus  $^{20}\text{Ne}$  and  $^{23}\text{Na}$  (in case of  $Z_1$  nuclei one excess neutron is glued to the single  $np$ -pair), and of the growing core consisting of the  $\alpha$ -clusters of the radius  $R_\alpha = 1.595$  fm [26]. The surface tension seems to be responsible for existence of core. It was shown that for the most stable nuclei the specific density of the core binding energy  $\rho$  is an approximately

constant value  $\rho = 2.55 \text{ MeV/fm}^3$  at the number of the surface molecule  $\alpha$ -clusters equal to three (three and a half with one excess neutron in case of odd  $Z_1$  nuclei). This provides an explanation of the particular number of excess neutrons in stable nuclei [27]. In the nuclei with smaller amount of excess neutrons the core is smaller (the molecule is bigger), because the number of neutrons provides the  $\rho$  (a little bit less than the saturated value) for a smaller number of  $\alpha$ -clusters. This theory allows one to calculate the radii for the nuclei with  $N \geq Z$  from analysis of the binding energy [26]. It explains the phenomena of a slight grow of the radii of the isotopes of one  $Z, Z_1$  with decreasing  $A, A_1$ . The obvious success of the model in describing binding energies and radii of the nuclei proves the validity of the EPN condition and its consequence (6).

The simplest way to obtain the value of LPPR in the cms of the nuclei  $A = 2Z$  and  $A_1 = 2Z_1$  is given by the formula

$$\begin{aligned} \Delta E_{np} &= \frac{(Z-1)e^2}{R_p^{A-1}} \\ \Delta E_{np1} &= \frac{(Z_1-1)e^2}{R_{p1}^{A_1-1}}. \end{aligned} \quad (9)$$

where  $R_p^{A-1}$  is the radius of the last proton position in the cms of the residual nucleus with the mass  $A-1$ . In case of  $R_p^{A-1} < R_C^{A-1}$  where  $R_C^{A-1} = r_C(A-1)^{1/3}$  the following equation is used (see (4))

$$\begin{aligned} \Delta E_{np} &= \frac{(Z-1)e^2}{2R_C^{A-1}} \left( 3 - \left( \frac{R_p^{A-1}}{R_C^{A-1}} \right)^2 \right) \\ \Delta E_{np1} &= \frac{(Z_1-1)e^2}{2R_{C1}^{A_1-1}} \left( 3 - \left( \frac{R_{p1}^{A_1-1}}{R_{C1}^{A_1-1}} \right)^2 \right), \end{aligned} \quad (10)$$

where  $r_C$  is obtained from the formula for the Coulomb energy  $E_C, E_{C1}$  (7) of the charge sphere with the radius  $R_C = r_C A^{1/3}$  and  $R_{C1} = r_{C1} A_1^{1/3}$

$$\begin{aligned} E_C &= 3/5 \frac{Z^2 e^2}{R_C} \\ E_{C1} &= 3/5 \frac{Z_1^2 e^2}{R_{C1}}. \end{aligned} \quad (11)$$

Values  $R_p$  and  $R_{p1}$  are related with  $R_p^{A-1}, R_{p1}^{A_1-1}$  as follows

$$\begin{aligned} R_p &= R_p^{A-1} \frac{(A-1)}{A} \\ R_{p1} &= R_{p1}^{A_1-1} \frac{(A_1-1)}{A_1}. \end{aligned} \quad (12)$$

The values  $R_{p,p1}$  obtained by means (9) and (10) are given in Table I. For the nuclei with  $Z, Z_1 \leq 8$  the values  $R_{p,p1}$  obtained by (10) are considerably less than  $R_{exp}$ , which is out of reason, and they are not presented in the table. (For example  $R_p = 2.209$  fm for  $^{16}\text{O}$  and

$R_{16\text{O}}=2.718$  fm [30]). For light nuclei the representation of nucleus as a charge sphere (11) is not good, therefore the values  $r_{C,C_1}$  do not give reasonable results to be used in (10).

The  $\alpha$ -cluster model also allows one to estimate the value of LPPR upon a suggestion that the Coulomb energy of the last proton (or  $np$ -pair) in even  $Z$  nucleus comes from the Coulomb interaction between two protons (two  $np$ -pairs) in the last  $\alpha$ -cluster  $\varepsilon_{\alpha}^C$  plus the Coulomb

TABLE I: The RLPP  $R_{p,p1}$  calculated in the framework of  $\alpha$ -cluster model representation.  $E^C$  is the Coulomb energy (7),  $r_C$  is the Coulomb radius (11),  $\Delta E_{np} = \varepsilon_n - \varepsilon_p$  [28]. The values  $R_{p,p1}^{(9)}$  and  $R_{p,p1}^{(10)}$  are obtained by means Eq.s (9) and (10) correspondingly with using (12), the values  $R_{p,p1}^{(13)}$  are calculated by (13) together with (15),  $R_{p,p1}^{(14)}$  are calculated by (14) with (16),  $R_{p,p1}^{aver}$  is calculated by (17).

$Z$	$E^C$	$r_C$	$\Delta E_{np}$	$R_{p,p1}^{(9)}$	$R_{p,p1}^{aver}$	$R_{p,p1}^{(10)}$	$R_{p,p1}^{(13)}$	$R_{p,p1}^{(14)}$
5	5.304	1.890	1.851	2.801	2.529		2.529	
6	8.067	1.684	2.763	2.389	2.491		2.491	
7	11.070	1.587	3.003	2.672	2.483		2.483	
8	14.607	1.502	3.537	2.672	2.764		2.764	
9	18.149	1.471	3.542	3.072	3.039	2.932	2.919	3.266
10	22.170	1.436	4.021	3.062	3.102	2.872	3.172	3.263
11	26.500	1.408	4.330	3.174	3.117	3.021	3.050	3.279
12	31.338	1.376	4.838	3.138	3.133	2.927	3.231	3.242
13	36.397	1.354	5.059	3.284	3.234	3.143	3.181	3.378
14	41.992	1.328	5.595	3.226	3.219	3.021	3.310	3.327
15	47.717	1.311	5.725	3.404	3.369	3.286	3.318	3.504
16	53.896	1.293	6.179	3.386	3.397	3.236	3.471	3.483
17	60.261	1.279	6.365	3.513	3.490	3.410	3.441	3.620
18	67.010	1.265	6.749	3.526	3.547	3.410	3.612	3.621
19	73.940	1.255	6.930	3.642	3.635	3.559	3.581	3.764
20	81.256	1.244	7.316	3.646	3.680	3.548	3.731	3.759
21	88.533	1.238	7.277	3.863	3.893	3.825	3.820	4.035
22	96.182	1.232	7.649	3.864	3.934	3.812	3.958	4.031
23	104.090	1.225	7.908	3.919	3.944	3.870	3.881	4.081
24	12.316	1.219	8.226	3.942	4.005	3.888	4.034	4.092
25	20.814	1.213	8.498	3.985	4.010	3.932	3.953	4.144
26	29.615	1.207	8.801	4.012	4.071	3.954	4.101	4.158
27	38.698	1.201	9.083	4.046	4.069	3.987	4.018	4.202
28	48.173	1.195	9.475	4.030	4.085	3.955	4.113	4.187
29	57.727	1.190	9.554	4.147	4.180	4.099	4.127	4.314
30	67.606	1.185	9.879	4.157	4.220	4.101	4.243	4.316
31	77.643	1.181	10.037	4.235	4.273	4.192	4.220	4.407
32	88.033	1.176	10.390	4.229	4.297	4.175	4.314	4.401
33	98.243	1.174	10.210	4.445	4.511	4.430	4.443	4.660
34	109.196	1.170	10.953	4.275	4.371	4.213	4.357	4.544
35	119.807	1.168	10.611	4.548	4.621	4.538	4.552	4.774
36	130.729	1.167	10.922	4.550	4.653	4.535	4.647	4.777
37	141.984	1.164	11.255	4.544	4.606	4.521	4.546	4.750
38	153.367	1.163	11.383	4.619	4.706	4.603	4.715	4.799
39	165.065	1.160	11.698	4.618	4.683	4.596	4.624	4.828
40	176.819	1.159	11.754	4.718	4.805	4.706	4.816	4.894
41	288.649	1.158	11.830	4.810	4.895	4.803	4.827	5.056
42	300.617	1.158	11.968	4.874	4.983	4.870	4.978	5.102
43	312.857	1.157	12.240	4.884	4.972	4.877	4.904	5.134
44	325.385	1.156	12.528	4.886	5.000	4.877	4.985	5.139
45	338.023	1.155	12.638	4.958	5.048	4.952	4.981	5.211

long range interaction of the last  $np$ -pair with the residual nucleus of the mass number  $A-4$ , consisting of  $N_{\alpha}-1$   $\alpha$ -clusters. For odd  $Z_1$  nucleus the energy of the last proton comes from the energy of the last  $np$ -pair interaction with the closest  $\alpha$ -cluster  $\varepsilon_{np\alpha}^C$  plus the energy of its Coulomb long range interaction with the residual nucleus of the mass number  $A-6$  [25]

$$\Delta E_{np} = \varepsilon_{\alpha}^C + \frac{(Z-2)e^2}{R_p^{A-4}}$$

$$\Delta E_{np1} = \varepsilon_{np\alpha}^C + \frac{(Z_1-3)e^2}{R_{p1}^{A_1-6}}, \quad (13)$$

where  $R_p^{A-4}$  and  $R_{p1}^{A_1-6}$  are the distances between the last  $np$ -pair and the cms of the residual nucleus with mass number  $A-4$  and  $A_1-6$ .

Another formula comes from the  $\alpha$ -cluster model representation that the last  $\alpha$ -cluster has three links with the nearest  $\alpha$ -clusters. Then the last  $\alpha$ -cluster Coulomb energy  $\Delta E_{\alpha}$  is equal to the sum of its own Coulomb energy  $\varepsilon_{\alpha}^C$ , the energy of the three links with the nearest clusters  $3\varepsilon_{\alpha\alpha}^C$  and the energy of the long range interaction with the rest  $\alpha$ -clusters of the nucleus  $2(Z-8)e^2/R_p^{A-16}$ , where  $R_p^{A-16}$  is the distance between the last proton and the cms of the remote  $\alpha$ -clusters with total mass  $A-16$ . In case of odd  $Z_1$  nucleus the Coulomb energy of the last  $np$ -pair is the sum of the Coulomb energy of its interaction with the three nearest  $\alpha$ -clusters  $3\varepsilon_{np\alpha}^C$  and the Coulomb energy of the long range interaction with the rest  $\alpha$ -clusters of the nucleus  $(Z_1-7)e^2/R_{p1}^{A_1-14}$ , where  $R_{p1}^{A_1-14}$  is the distance between the last proton and the cms of the remote  $\alpha$ -clusters with the total mass number  $A_1-14$  [25]

$$\Delta E_{\alpha} = \varepsilon_{\alpha}^C + 3\varepsilon_{\alpha\alpha}^C + \frac{2(Z-8)e^2}{R_p^{A-16}}$$

$$\Delta E_{np1} = 3\varepsilon_{np\alpha}^C + \frac{(Z_1-7)e^2}{R_{p1}^{A_1-14}}, \quad (14)$$

where  $\Delta E_{\alpha} = \Delta E_{np1} + \Delta E_{np}$ . The simplest formula for the Coulomb energy decreasing with distance is used, as the values  $R_p^{A-4}$ ,  $R_{p1}^{A_1-6}$ ,  $R_p^{A-16}$  and  $R_{p1}^{A_1-14}$  are surely bigger than the nuclear radius.

The distance between the cms of the last  $\alpha$ -cluster and the cms of the residual nucleus with mass number  $A-4$  is estimated as  $R_p^{A-4} - R_{4\text{He}}$ . Then  $(R_p^{A-4} - R_{4\text{He}})(A-4)/A$  is the radius of the last  $\alpha$ -cluster position in the cms of the nucleus  $A$ . The same logic is used for the odd  $Z_1$  nucleus.  $(R_{p1}^{A_1-6} - R_{4\text{He}+np})(A_1-6)/A_1$  is the radius of the position of the last cluster consisting of the 1.5  $\alpha$ -clusters in the cms of nucleus  $A_1$  and  $R_{4\text{He}+np} = R_{4\text{He}}1.5^{1/3}$  (8). So LPPRs in cms of nucleus  $A$   $R_{p,p1}$  are calculated as follows

$$R_p = (R_p^{A-4} - R_{4\text{He}}) \frac{A-4}{A} + R_{4\text{He}}$$

$$R_{p1} = (R_{p1}^{A_1-6} - R_{4\text{He}+np}) \frac{A_1-6}{A_1} + R_{4\text{He}+np}. \quad (15)$$

To calculate  $R_{p,p1}$  in case of (14) one can use LPPR in the nuclei  $^{16}\text{O}$   $R_p^{16}$  and  $^{14}\text{N}$   $R_{p1}^{14}$  obtained by (9), see Table I. Then using the same logic as in (15) we have

$$\begin{aligned} R_p &= (R_p^{A-16} - R_p^{16}) \frac{A-16}{A} + R_p^{16} \\ R_{p1} &= (R_{p1}^{A_1-14} - R_{p1}^{14}) \frac{A_1-14}{A_1} + R_{p1}^{14}. \end{aligned} \quad (16)$$

Average value  $R_{p,p1}^{aver}$  of the radii estimated with using the  $\alpha$ -cluster model parameters  $r_C$ ,  $\varepsilon_\alpha^C$ ,  $\varepsilon_{\alpha\alpha}^C$ ,  $\varepsilon_{np\alpha}^C$  is also calculated as follows

$$R_{p,p1}^{aver} = (R_{p,p1}^{(10)} + R_{p,p1}^{(13)} + R_{p,p1}^{(14)})/3. \quad (17)$$

The values LPPR in cmc of nuclei  $A, A_1$  estimated by different ways are given with upper indexes corresponding to the Eqs, see Table I. One can see from the table that the values  $R_{p,p1}^{(9)}$  obtained without model parameters but with the assumption (6), which is the straight consequence of the EPN condition, are in agreement with  $R_{p,p1}^{aver}$  with the average deviation in 0.065 fm. For some of the light and the heaviest nuclei the difference  $|R_{p,p1}^{(9)} - R_{p,p1}^{aver}|$  is more than 0.1 fm.

Let us check if the values  $R_{p,p1}^{(9)}$  and  $R_{p,p1}^{aver}$  are consistent with the experimental radii. Not for all symmetrical nuclei there are experimental data. In those cases the experimental radius of the nearest isotope is used [32]. In the case of no data, this is the case of  $Z_1=43$ , Eq. (8) is used. The squared nuclear radius  $((R^{A,Z})^2, (R^{A_1,Z_1})^2)$  is calculated as the sum of the squared radius of the residual nucleus  $((R^{A-1,Z-1})^2, (R^{A_1-1,Z_1-1})^2)$  and the square radius of the last proton position in the cms of the residual nucleus  $((R_p^{A-1})^2, (R_{p1}^{A_1-1})^2)$  weighed

$$\begin{aligned} (R^{A,Z})^2 &= \frac{(Z-1)}{Z} (R^{A-1,Z-1})^2 + \frac{1}{Z} (R_p^{A-1})^2 \\ (R^{A_1,Z_1})^2 &= \frac{(Z_1-1)}{Z_1} (R^{A_1-1,Z_1-1})^2 + \frac{1}{Z_1} (R_{p1}^{A_1-1})^2 \end{aligned} \quad (18)$$

With growing  $Z, Z_1$  the difference  $|R_{p,p1}^{(9)} - R_{p,p1}^{aver}|$  does not affect much the calculated nuclear radii (18). The average deviation  $|R_{exp} - R_{th}|$  where  $R_{th} = R^{A,Z}, R^{A_1,Z_1}$  is 0.058 fm for  $R_{p,p1}^{(9)}$  and 0.056 fm for  $R_{p,p1}^{aver}$ . For the nuclei with  $Z, Z_1 = 5, 6, 7, 8, 9, 10, 11$  the difference  $|R_{p,p1}^{(9)} - R_{p,p1}^{aver}| = 0.272, 0.102, 0.189, 0.092, 0.033, 0.040, 0.057$  fm leads to the corresponding differences in the values  $|R_{exp} - R_{th}|$  calculated with using  $R_{p,p1}^{(9)}$  and  $R_{p,p1}^{aver}$  0.069, 0.002, 0.032, 0.001, 0.005, 0.005, 0.006 fm, which shows that using  $R_{p,p1}^{(9)}$  and  $R_{p,p1}^{aver}$  give close values of  $R_{th}$ . It should be noticed here that for the nuclei with  $Z, Z_1 = 5, 6, 8, 10, 11$   $R_{p,p1}^{aver}$  gives better agreement with the experimental data. For the nuclei with  $Z, Z_1 \leq 8$  the values  $R_{p,p1}^{aver} = R_{p,p1}^{(13)}$  (see Table I). It is obvious that in the case of the light nuclei the values  $R_{p,p1}^{(13)}$  are more preferable than  $R_{p,p1}^{(9)}$ , but the obvious advantage of the

latter is that  $R_{p,p1}^{(9)}$  are obtained without any parameters only upon the suggestion (6). For further calculations the values LPPR  $R_{p,p1}^{(9)}$  are used and they are indicated as  $R_{p,p1}$ .

### III. BOUND STATE POTENTIAL PARAMETERS IN SYMMETRICAL NUCLEUS

Having obtained LPPR  $R_{p,p1}$  one can numerically calculate the radius  $R_{Cp} = r_C^{DWBA} (A-1)^{1/3}$  and  $R_{Cp1} = r_{C1}^{DWBA} (A_1-1)^{1/3}$  from (4) at  $E_{Cp} = \Delta E_{np}$  and  $r = R_{p,p1}$ . The obtained values  $r_{C,C1}^{DWBA}$  are used to solve the Shrödinger equation for one nucleon bound state with the W-S potential with the spin-orbit part of Thomas form with  $\lambda=25$  as it is used in DWUCK program [3]

$$V(r) = V_0(f(r) + \frac{\lambda}{45.2} \frac{1}{r} \frac{df(r)}{dr} \vec{L} \vec{\sigma}) \quad (19)$$

where

$$f(r) = [1 + \exp(\frac{r - r_0(A-1)^{1/3}}{a_0})]^{-1}. \quad (20)$$

The EPN-condition used together with the well-depth procedure makes the parameter  $r_{C,C1}^{DWBA}$  the critical one in determination of the last proton rms radius  $\langle r_{p,p1}^2 \rangle$  and AC  $b_{n,p}$  [13, 14]. Variation one of the parameters at the  $r_{C,C1}^{DWBA}$  fixed, for example the diffuseness  $a_0 = 0.4 \div 0.7$  fm, changes the other parameters  $r_0$  and  $V_0$  that way that  $\langle r_{n/p}^2 \rangle^{1/2}$  stays almost the same (variety within 1%) and AC  $b_{n/p}$  changes within a few per cent [14]. So one can say that in such calculations the Coulomb radius defines last proton rms radius. Quantum numbers  $n, l, j$  are selected according to the Pauli conservation principle, parity conservation rule and the sum rule for momenta. In Table II for the cases when both binding energies and spins of nuclei  $A$  and  $B$  are known [28], the values  $r_{C,C1}^{DWBA}$  and corresponding values  $r_0$  and  $V_0$  are given together with the rms radii for neutron/proton  $\langle r_{n/p}^2 \rangle^{1/2}$  and AC  $b_{n/p}$ .

Here  $a_0=0.65$  fm is used. The value  $a_0=0.55$  fm in the case of nucleus  $^{24}\text{Mg}$  gives  $r_0=1.2592$  fm  $V_0=60.0168$  MeV  $\langle r_n^2 \rangle^{1/2}=3.160$  fm,  $\langle r_p^2 \rangle^{1/2}=3.210$  fm,  $b_n=8.561$  fm $^{-1/2}$  and  $b_p=12.54$  fm $^{-1/2}$ . Comparison of the values with the data in Table II shows that  $a_0=0.55$  fm almost does not change rms radius and AC changes for 6%. Searching parameters under the EPN condition is done by a special program with using iteration. Result may depend on the starting values, which shows the accuracy of the calculation. This is few 0.001 fm for  $r_0$  and few 0.01 MeV for  $V_0$ , which consequences the rms radius accuracy of 0.001 fm and the AC accuracy of few 0.01 fm $^{-1/2}$ . For example, in case of the nucleus Mg the result does not depend on the starting values. For the nucleus  $^{44}\text{Ti}$  another solution gives  $r_0=1.2719$  fm and  $V_0=-60.8539$  MeV,  $\langle r_p^2 \rangle^{1/2}=3.962$  fm and  $b_p = 31.43$  fm $^{-1/2}$ .

TABLE II: Parameters of the W-S potential at the EPN-condition. In the first column the bound state  $A = B + n/p$ , in the 2nd and the 3d columns there are the quantum numbers  $n, l, j$  and experimental binding energies  $\varepsilon_{n,p}$ , the 4th column gives Coulomb radii  $r_{C,C1}^{DWBA}$  (4), the 5th and the 6th contains the parameters  $r_0$  and  $V_0$  at  $a_0 = 0.65$  fm and rms radii  $\langle r_{n,p}^2 \rangle^{1/2}$ , the 7th column contains  $R_{p,p1}$  (see Table I), the 8th column presents AC  $b_{n,p}$ , the 9th contains AC  $b_{n,p}^{st}$  calculated at the standard parameters, the 10th column gives the error brought by the standard parameters in DWBA analysis to obtain SF (21).

$A$ $B + n/p$	$n, l, j$	$\varepsilon_{n,p}$ MeV	$r_{C,C1}^{DWBA}$ fm	$r_0$ fm $V_0, \text{MeV}$	$\langle r_{n,p}^2 \rangle^{1/2}$ fm	$R_{p,p1}^{aver}$ fm	$b_{n,p}$ $\text{fm}^{-1/2}$	$b_{n,p}^{st}$ $\text{fm}^{-1/2}$	err %
$^{10}\text{B} = ^9\text{B} + n$	1,1,3/2	8.438		1.2198	2.845		2.893	2.961	5
$^{10}\text{B} = ^9\text{Be} + p$		6.587	1.8466	-52.5783	2.888	2.801	3.212	3.306	6
$^{12}\text{C} = ^{11}\text{C} + n$	1,1,3/2	18.720		1.1074	2.418		7.411	8.807	41
$^{12}\text{C} = ^{11}\text{B} + p$		15.957	1.4249	-72.2489	2.445	2.389	8.461	10.09	42
$^{14}\text{N} = ^{13}\text{N} + n$	1,1,1/2	10.554		1.1592	2.716		3.877	4.230	19
$^{14}\text{N} = ^{13}\text{C} + p$		7.550	1.4702	-60.5385	2.777	2.672	4.587	5.024	20
$^{16}\text{O} = ^{15}\text{O} + n$	1,1,1/2	15.663		1.2784	2.708		7.860	7.578	7
$^{16}\text{O} = ^{15}\text{N} + p$		12.128	1.3746	-56.4093	2.757	2.672	9.784	9.468	7
$^{18}\text{F} = ^{17}\text{F} + n$	1,2,5/2	9.149		1.1488	3.106		2.403	2.757	32
$^{18}\text{F} = ^{17}\text{O} + p$		5.607	1.4919	-66.9201	3.167	3.072	2.937	3.383	33
$^{20}\text{Ne} = ^{19}\text{Ne} + n$	1,0,1/2	16.865		1.2228	3.106		20.11	20.86	8
$^{20}\text{Ne} = ^{19}\text{F} + p$		12.844	1.4142	-78.0952	3.162	3.062	28.65	29.81	8
$^{22}\text{Na} = ^{21}\text{Na} + n$	1,2,5/2	11.069		1.1881	3.205		3.922	4.321	21
$^{22}\text{Na} = ^{21}\text{Ne} + p$		6.740	1.4031	-60.4515	3.272	3.174	5.434	6.010	22
$^{24}\text{Mg} = ^{23}\text{Mg} + n$	1,2,3/2	16.531		1.2300	3.162		9.085	7.846	25
$^{24}\text{Mg} = ^{23}\text{Na} + p$		11.693	1.3320	-63.3681	3.216	3.138	13.36	11.59	25
$^{26}\text{Al} = ^{25}\text{Al} + n$	1,2,5/2	11.365		1.1954	3.312		4.695	5.148	20
$^{26}\text{Al} = ^{25}\text{Mg} + p$		6.306	1.3454	-55.5656	3.394	3.284	7.369	8.103	21
$^{28}\text{Si} = ^{27}\text{Si} + n$	1,2,5/2	17.180		1.2299	3.248		11.26	11.73	9
$^{28}\text{Si} = ^{27}\text{Al} + p$		11.585	1.2787	-60.0035	3.313	3.226	18.41	19.20	9
$^{30}\text{P} = ^{29}\text{P} + n$	1,2,3/2	11.319		1.3105	3.445		5.487	4.917	20
$^{30}\text{P} = ^{29}\text{Si} + p$		5.595	1.3085	-52.7940	3.551	3.404	10.05	9.036	19
$^{32}\text{S} = ^{31}\text{S} + n$	2,0,1/2	15.042		1.2843	3.464		25.72	24.31	11
$^{32}\text{S} = ^{31}\text{P} + p$		8.863	1.2660	-56.2844	3.585	3.386	53.65	50.84	10
$^{34}\text{Cl} = ^{33}\text{S} + n$	1,2,3/2	11.508		1.3172	3.552		6.407	5.631	23
$^{34}\text{Cl} = ^{33}\text{S} + p$		5.143	1.2795	-48.9613	3.675	3.513	14.02	12.35	22
$^{36}\text{Ar} = ^{35}\text{Ar} + n$	1,2,3/2	15.256		1.3522	3.543		12.45	9.923	36
$^{36}\text{Ar} = ^{35}\text{Cl} + p$		8.507	1.2534	-50.8395	3.645	3.526	27.33	21.86	36
$^{38}\text{K} = ^{37}\text{K} + n$	1,2,3/2	12.072		1.3385	3.666		8.117	6.771	30
$^{38}\text{K} = ^{37}\text{Ar} + p$		5.143	1.2646	-45.6836	3.802	3.642	21.50	17.96	30
$^{40}\text{Ca} = ^{39}\text{Ca} + n$	1,2,3/2	15.644		1.3692	3.659		15.31	11.59	43
$^{40}\text{Ca} = ^{39}\text{K} + p$		8.328	1.2394	-47.8702	3.775	3.646	39.52	29.93	43
$^{42}\text{Sc} = ^{41}\text{Sc} + n$	1,3,7/2	11.550		1.2399	3.888		4.526	4.636	5
$^{42}\text{Sc} = ^{41}\text{Ca} + p$		4.273	1.2868	-57.6593	3.992	3.863	13.09	13.43	5
$^{44}\text{Ti} = ^{43}\text{Ti} + n$	1,3,7/2	16.299		1.2713	3.874		11.52	10.87	11
$^{44}\text{Ti} = ^{43}\text{Sc} + p$		8.650	1.2616	-60.8911	3.961	3.864	31.37	29.63	11
$^{46}\text{V} = ^{45}\text{V} + n$	1,3,7/2	13.265		1.2485	3.941		6.994	7.021	1
$^{46}\text{V} = ^{45}\text{Ti} + p$		5.357	1.2564	-56.9517	4.048	3.919	23.13	23.22	1
$^{48}\text{Cr} = ^{47}\text{Cr} + n$	2,1,3/2	16.332		1.3556	4.022		51.62	40.74	38
$^{48}\text{Cr} = ^{47}\text{V} + p$		8.106	1.2422	-60.0458	4.170	3.942	200.1	159.5	36
$^{50}\text{Mn} = ^{49}\text{Mn} + n$	1,3,5/2	13.083		1.4531	3.991		7.332	9.748	77
$^{50}\text{Mn} = ^{49}\text{Cr} + p$		4.585	1.2352	-54.0698	4.114	3.985	31.76	35.79	27
$^{52}\text{Fe} = ^{51}\text{Fe} + n$	1,3,5/2	16.180		1.3608	4.017		13.97	10.04	48
$^{52}\text{Fe} = ^{51}\text{Mn} + p$		7.379	1.2232	-58.3261	4.129	4.012	55.20	39.60	49
$^{54}\text{Co} = ^{53}\text{Co} + n$	1,3,7/2	13.436		1.3275	4.276		10.80	8.740	35
$^{54}\text{Co} = ^{53}\text{Fe} + p$		4.353	1.2145	-48.8366	4.426	4.062	71.25	57.64	35
$^{56}\text{Ni} = ^{55}\text{Ni} + n$	1,3,7/2	16.639		1.2430	4.032		15.21	15.53	4
$^{56}\text{Ni} = ^{55}\text{Co} + p$		7.164	1.1928	-56.6589	4.150	4.030	71.57	72.70	3
$^{58}\text{Cu} = ^{57}\text{Cu} + n$	2,1,3/2	12.423		1.2895	4.212		30.42	28.00	15
$^{58}\text{Cu} = ^{57}\text{Ni} + p$		2.869	1.2103	-53.3181	4.472	4.147	499.6	462.1	14
$^{60}\text{Zn} = ^{59}\text{Zn} + n$	2,1,3/2	14.998		1.3652	4.264		55.93	42.60	42
$^{60}\text{Zn} = ^{59}\text{Cu} + p$		5.119	1.1965	-52.2073	4.476	4.157	524.5	405.5	40
$^{62}\text{Ga} = ^{61}\text{Ga} + n$	2,1,3/2	12.981		1.4849	4.598		55.21	32.56	65
$^{62}\text{Ga} = ^{61}\text{Zn} + p$		2.944	1.2028	-43.7735	4.878	4.235	1423.	1877.	74
$^{64}\text{Ge} = ^{63}\text{Ge} + n$	2,1,3/2	15.482		1.3899	4.346		69.04	48.82	50
$^{64}\text{Ge} = ^{63}\text{Ga} + p$		5.092	1.1865	-50.2869	4.564	4.229	820.5	592.1	48

$A$ $B + n/p$	$n, l, j$	$\varepsilon_{n,p}$ MeV	$r_{C,C1}^{DWBA}$ fm	$r_0$ fm $V_0, \text{MeV}$	$\langle r_{n,p}^2 \rangle^{1/2}$ fm	$R_{p,p1}$ fm	$b_{n,p}$ $\text{fm}^{-1/2}$	$b_{n,p}^{st}$ $\text{fm}^{-1/2}$	err %
$^{68}\text{Se} = ^{67}\text{Se} + n$	1,3,5/2	15.809		1.3346	4.266		17.98	13.67	42
$^{68}\text{Se} = ^{67}\text{As} + p$		4.856	1.1705	-51.6985	4.429	4.275	217.9	164.0	43
$^{72}\text{Kr} = ^{73}\text{Kr} + n$	1,3,5/2	15.089		1.4714	4.726		27.04	13.18	76
$^{72}\text{Kr} = ^{73}\text{Br} + p$		4.167	1.2186	-42.9041	4.934	4.550	668.5	324.5	76
$^{74}\text{Rb} = ^{73}\text{Rb} + n$	1,3,5/2	13.910		1.3744	4.530		16.62	11.19	55
$^{74}\text{Rb} = ^{73}\text{Kr} + p$		2.655	1.2035	-45.0682	4.730	4.544	864.0	579.8	55
$^{76}\text{Sr} = ^{75}\text{Y} + n$	2,1,3/2	15.694		1.5215	4.760		127.2	60.91	77
$^{76}\text{Sr} = ^{75}\text{Rb} + p$		4.311	1.2108	-42.3425	4.999	4.619	4304.	2173.	75
$^{78}\text{Y} = ^{77}\text{Y} + n$	2,2,5/2	13.744		1.3316	4.688		47.23	37.79	36
$^{78}\text{Y} = ^{77}\text{Sr} + p$		2.046	1.1981	-58.8944	4.946	4.618	9098.	7384.	34

The procedure of seeking parameters under the EPN-condition brings the determination of rms radius  $\langle r_{p,p1}^2 \rangle^{1/2}$  and radius  $r_0$  by the Coulomb radius  $r_{C,C1}^{DWBA}$  in a way that decreasing the latter causes increasing the values  $\langle r_{p,p1}^2 \rangle^{1/2}$  and  $r_0$ .

In the table also the values AC  $b_{n,p}^{st}$  are given which have been calculated at the standard parameters  $r_0=1.25$  fm,  $a_0=0.65$  fm with  $r_{C,C1}^{DWBA}=1.25$  fm at well-depth procedure without addition of the EPN condition. One wants to know what error in obtaining SF in analysis of one nucleon transfer reactions by means of DWBA is brought with using standard parameters. In case of the pure peripheral reactions the cross sections at main scattering peak contains an asymptotic part of the wave function (1) and (2). Then the deviation of the SF  $S$  extracted by the DWBA analysis with standard parameters from the value obtained with using the EPN-parameters is calculated as follows

$$err = \frac{|b_{n/p}^2 - (b_{n/p}^{st})^2|}{b_{n/p}^2} * 100\% \quad (21)$$

The values  $R_{p,p1}^{aver}$  (see Table I) for the light nuclei have also been tried in obtaining the parameters. In this case the proton rms radii are also in agreement with the values  $R_{p,p1}^{aver}$  within 0.1 fm. For the nuclei with  $Z, Z_1 = 5, 6, 7, 8, 9, 10, 11, 12$  the parameter  $r_{C,C1}^{DWBA}=1.955$  fm, 1.360 fm, 1.553 fm, 1.312 fm, 1.510 fm, 1.389 fm, 1.434 fm, 1.335 fm, which brings the  $r_0, V_0 = 0.88$  fm, -79 MeV; 1.24 fm, -64 MeV; 0.93 fm, -87 MeV; 1.38 fm, -50 MeV, 1.12 fm, -69 MeV; 1.28 fm, -51 MeV; 1.14 fm, -64 MeV; 1.23 fm, -64 MeV. This means that the parameters given in Table II should be taken with the probability corresponding to the deviation between the results of using  $R_{p,p1}$  and  $R_{p,p1}^{aver}$ , see Table III. It is seen from the table that deviation in  $r_{C,C1}^{DWBA}$  is smaller for bigger  $Z, Z_1$ , so is the deviation in the error. In case of the nuclei  $^{10}\text{B}$ ,  $^{12}\text{C}$ ,  $^8\text{O}$  and  $^{10}\text{Ne}$  the smallest possible error in obtaining SF with standard parameters in case of pure peripheral reactions is small, which is 5, 3, 7, 7. But if the proper parameters differ in full length of the possible deviation the error becomes significant, except for  $^{10}\text{Ne}$ .

In the calculations the value  $b_n^2/b_p^2$  obtained under the EPN condition is almost stable value to a wide variety of

TABLE III: Deviations in results of searching parameters with using  $R_{p,p1}$  and  $R_{p,p1}^{aver}$ . The indications are the same as in Table II.

$Z$	$r_{C,C1}^{DWBA}$	$r_0$ fm	$b_p$	err%
5	1.847÷1.955	0.883÷1.220	2.51÷3.21	6÷72
6	1.360÷1.425	1.109÷1.240	8.47÷9.94	3÷42
7	1.470÷1.553	0.930÷1.161	3.76÷4.59	20÷79
8	1.312÷1.375	1.279÷1.384	9.78÷11.3	7÷29
9	1.492÷1.510	1.120÷1.151	2.83÷2.94	33÷43
10	1.389÷1.414	1.223÷1.281	28.7÷31.0	7÷8
11	1.403÷1.434	1.144÷1.188	5.07÷5.43	22÷41
12	1.332÷1.335	1.226÷1.230	13.3÷13.4	25÷25

$r_{C,C1}^{DWBA}$ . Varying  $r_{C,C1}^{DWBA}=1.1-1.5$  fm brings a change of the ratio within 2 %, which can be used to predict the ratio of experimentally obtained values ANC's  $C_n^2/C_p^2$  due to the equality of the spectroscopic factors of symmetrical bound states.

#### IV. DISCUSSION OF RESULTS

In FIG. 1 the graphs of the values of different kinds of radii in dependence on  $Z$  are given (in this section for further convenience even  $Z$  and odd  $Z_1$  are indicated as  $Z$ , as well as the other values like  $A$ ,  $R_p$ ,  $R_C$  and so on) [t]

These are the Coulomb radius  $R_C = r_C A^{1/3}$  (11) of the sphere having the Coulomb energy  $E_C^C$  (7), the values of LPPR  $R_p$ , the values  $R_C^{DWBA} = r_C^{DWBA} A^{1/3}$  where  $r_C^{DWBA}$  is obtained by (4) with using  $R_p$ , rms radius for the last proton  $\langle r_p^2 \rangle^{1/2}$  calculated with the wave function obtained with using the EPN parameters for nuclear potential determined by the parameter  $r_C^{DWBA}$ . Also the values of the radius of the nuclear potential  $R_0 = r_0 A^{1/3}$  (for  $r_0$  see Table II) and for the standard values  $r_0 = r_C^{DWBA}=1.25$  fm in comparison with experimental nuclear radii  $R_{exp}$  are given. The nuclear radii  $R^{(A,Z)}$  are calculated by (18) where instead of  $R_p^{A-1}$  the proton rms radii transferred into the cms of the nucleus  $A-1$  by (12) are used.

Comparison of the values  $R_C$  and  $R_C^{DWBA}$  shows that

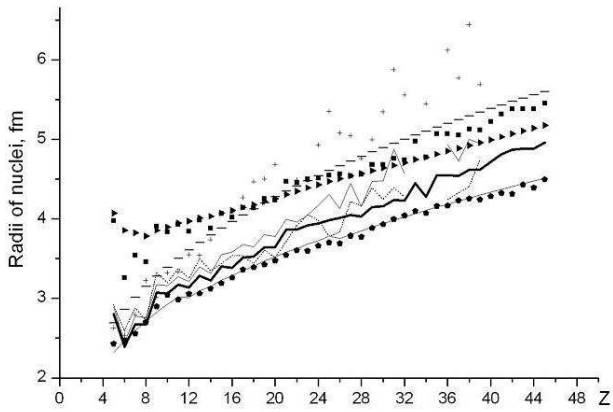


FIG. 1: Nuclear radii. Symbols indicate the following values: the experimental nuclear radii  $R_{exp}$  (pentagons) [28] (in case of absent data for the symmetrical nuclei the radius of the nearest isotope is taken, for the nucleus with  $Z=43$ , which is not presented in [28], the radius is calculated by (8)), the Coulomb radius  $R_C$  (11) (triangles), the radius  $R_C^{DWBA} = r_C^{DWBA} A^{1/3}$  (4) (squares) and the radius of nuclear potential  $R_0 = r_0 A^{1/3}$  satisfying the EPN-condition (crosses). Lines indicate: RLPP  $R_p$  (solid thick line), rms radii  $\langle r_p^2 \rangle^{1/2}$  (thin line), calculated nuclear radius  $R^{A,Z}$  (18) (solid thin line going along the symbols of  $R_{exp}$ ) (in Eq. (18) instead  $R_p^{A-1}$  the values rms radii  $\langle r_p^2 \rangle^{1/2}$  transferred into the c.m.s. of the nucleus  $A-1$  by (12) are used). Also the standard potential radius  $R_0 = 1.25 A^{1/3}$  (dashed line) and the corresponding rms radii  $\langle r_p^2 \rangle^{1/2}$  (line of dots) are given.

they are in agreement for the nuclei with  $Z=5$  and  $9 \leq Z \leq 32$  with the average deviation in 0.12 fm. The reason of the considerable difference between the values for the light nuclei  $Z < 9$  is that the approach of a charge sphere is not good for the case. That is one of the reasons why the value  $r_C$  is not used here to obtain the parameters. The other reason is that for the nuclei with  $Z > 32$  the values  $r_C < r_C^{DWBA}$ , so the corresponding potential has either too big radius or in some cases there are no parameters satisfying the EPN-condition.

In FIG. 2 the Coulomb energy  $E^C$  (7) of the charge sphere of radius  $R_C$  is given in comparison with the energies of two charge spheres with the radii equal to  $R_C^{DWBA}(R_p)$  and  $R_C^{DWBA}(R_p^{aver})$  obtained by (4) with using values  $R_p$  and  $R_p^{aver}$ . For the nuclei with  $5 \leq Z \leq 26$  all three lines deviate within few MeV. For heavier nuclei they fork. The Coulomb energy of the sphere of the radius  $R_C^{DWBA}(R_p^{aver})$  is the highest graph and it splits from the others at  $Z \geq 26$ , and these two deviate at  $Z \geq 32$ .  $E^C$  is the middle line between the Coulomb energies of the two radii spheres. For the heavier nuclei the deviation grows within a corridor  $E^C \pm 15$ . For comparison the Coulomb energy according to the Weizsäcker formula [29] is given too, which is in agreement with the first three graphs within average deviation in few MeV for the nuclei  $5 \leq Z \leq 32$  and for heavier nuclei it is in agreement with the energy of the sphere of radius  $R_C^{DWBA}(R_p)$ .

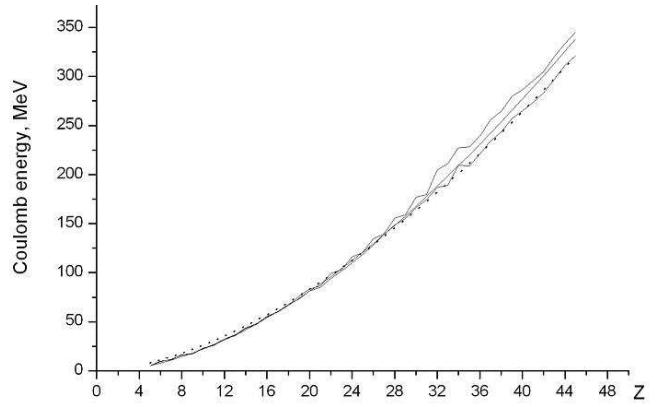


FIG. 2: The Coulomb energies of two charge spheres having the radii  $R_C^{DWBA}$  calculated by (4) with  $R_p$  (lower line) and  $R_C^{DWBA} = r_C^{DWBA} A^{1/3}$  calculated with  $R_p^{aver}$  (upper line) in comparison with the Coulomb energy  $E_C$  (7) (middle line). The Coulomb energy due to the Weizsäcker formula [29] is indicated with dotted line.

The mean field model considers a nucleus as consists of neutrons and protons moving independently in a mean field. The question about consistency of the W-S potential used as a single particle potential with experimental data on nuclear charge and matter distributions was discussed in a number of articles, for example [2, 7–9]. There was shown that theoretical charge distribution calculated with using the sum of squared single particle wave functions produced by the W-S potential with a radius close to the standard one is in agreement with experimental charge density distribution for the nuclei with large deviation of  $A$ . But the sizes of internal shells are not known, so the fitting experimental radius can be done by varying sizes of internal shells. Besides, as it is already shown in section II experimental radius alone can't be a sensitive criterion for checking validity of the last proton rms radii calculated at the parameters.

The values LPPR obtained from independent analysis of nuclear binding energies allow one to check the rms radius  $\langle r_p^2 \rangle^{1/2}$  for the last proton produced by the W-S potential. From FIG. 1 (see also Table II) one can see that for the nuclei with  $Z \leq 14$  the EPN potential provides a good description  $R_p$ , the difference is  $\sim 0.1$  fm. For the other nuclei with  $Z > 14$  the difference  $\langle r_p^2 \rangle^{1/2} - R_p$  is larger, more than 0.2 fm. It leads one to a conclusion that the W-S potential as it is used in the single particle potential model is not good for the heavier nuclei. The values LPPR have also been tried here to obtain the Coulomb radius  $r_C^{DWBA}$  from requirement of equivalence of  $R_p = \langle r_p^2 \rangle^{1/2}$  (first it was proposed in [15] for some of the nuclei of 1d shell but was not fulfilled correctly, because the values of LPPR were taken in the c.m.s of the residual nucleus). In that case for heavier nuclei the value  $r_C^{DWBA}$  becomes unreasonably big or there is no solution of the Schrödinger equation at all, which leads to the same conclusion.



In spite of the fact that the proton rms radii for the nuclei with  $Z > 14$  considerably deviate from the  $R_p$  the nuclear radii  $R^{(A,Z)}$  (18) with using  $\langle r_p^2 \rangle^{1/2}$  (re-calculated (12) for the cms of the residual nucleus with mass number  $A-1$ ) are consistent with  $R_{exp}$ , see FIG.1. That is because the relative weight of the last proton decreases with growing  $Z$ . This is another evidence that the experimental nuclear radius alone can not be used as a sensitive test for validity of the last proton potential parameters without an additional criterion like  $R_p$ .

Another proof of unavailability of the W-S potential is the size of the nuclear potential well. The potential radius  $R_0$  grows with  $Z$  by a periodic function, see FIG. 1. The values  $R_0$  become too big in comparison with the experimental radii. For heavier nuclei the difference grows up to 2 fm, which is not right taking into account the short range nuclear force. For some nuclei the radius descends to the standard value  $1.25A^{1/3}$ , which is also too big in comparison with the experimental radius (the explanation is in next paragraph). One also should notice here that for the nuclei with  $Z \geq 15$  radius  $R_0$  increases by a leap. Searching parameters under the EPN condition for the nucleus  $^{30}\text{P}$  gives  $V_0 = 52.78$  at  $R_0 = 4.07$  fm at  $r_0 = 1.31$  fm, see Table II. Thus, one can suppose that the number of nuclei for which the W-S potential is good is restricted by those with  $Z \leq 14$ . For the nuclei with  $Z \geq 15$  the radius  $R_0$  obtained at the EPN condition growing with a leap reveals that the W-S potential is not proper in the framework of single particle representation.

Let us consider the consistency of the W-S potential with experimental charge radii at the standard parameters used without the EPN condition. The difference between LPPR  $R_p$  and  $\langle r_p^2 \rangle^{1/2}$  at the standard parameters for the nuclei with  $Z < 14$  is bigger than that for the parameters of the EPN condition, see FIG. 1. For the odd  $Z$  nuclei it exceeds 0.2 fm. For the nuclei with  $Z \geq 14$  the rms radii are in the better agreement with LPPR. But then the potential radius  $1.25A^{1/3}$  is not consistent with the experimental radii. Experimental radius  $1.005A^{1/3} < R_{exp} < 1.077A^{1/3}$  for the nuclei with  $10 < Z \leq 24$  and  $R_{exp} = 1.005A^{1/3}$  for the other nuclei with  $Z > 24$ , see Eq. (8) rewritten for  $A = 4N_\alpha$ . It means that the nuclear potential radius  $1.25A^{1/3}$  grows with  $Z$  with a bigger rate than the radius of the nucleus, which is in a clear discrepancy with the short range nuclear force. One can see in FIG. 1 that the Coulomb radius  $R_C$  obtained in the framework of the  $\alpha$ -cluster model relates with  $R_{exp}$  as  $R_C = R_{exp} + d$  where  $d$  is an approximately constant value. Potential radius  $R_0$  can be equal to  $R_C$  or, if it is different, at least it should grow with  $Z$  in consistency with  $R_C$  and with  $R_{exp}$ . So one can suppose that the relation between  $R_0$  and  $R_{exp}$  should be like  $R_0 = R_{exp} + d$ , which is consistent with the nature of short range nuclear force.

Besides, at standard parameters the neutron and proton potential wells are allowed to be different. At  $r_0 = 1.25$  fm in case of the nucleus  $^{30}\text{P}$  neutron's potential depth is less than the proton's one,  $V_{0n} = 57.15$  MeV

and  $V_{0p} = 57.50$ . The difference does not seem significant in point of view of mean field theory where neutrons and protons are distributed independently and their centers of mass do not coincide. But it is not right in the representation of the  $\alpha$ -cluster model, where a nucleus consists of  $np$ -pairs joined in  $\alpha$ -clusters and the position of the last neutron and the last proton belonging to one pair is determined by one potential. Calculations show that at the EPN condition the proton stays farther from the center of mass of a nucleus  $A$  than the neutron  $\langle r_p^2 \rangle^{1/2} > \langle r_n^2 \rangle^{1/2}$  (see Table II), which also appears natural. If one takes into account the difference in neutron and proton masses, the neutron potential is supposed to be deeper than proton's one, but not the opposite. It should be pointed out that the standard potential radius  $1.25A^{1/3}$  is less than the potential radius  $R_0$  obtained at the EPN condition only because at the standard parameters the neutron and proton depths are allowed to be different at unreasonable prevalence of proton's nuclear potential above neutron's one. Thus, one can see here that a good description of experimental charge radii in the framework of single particle potential model is provided by an unreasonably big radius of the potential at both standard parameters and at the parameters of the EPN condition. It is not so big as one at the EPN condition, because the neutron and proton potential wells are allowed to be different in the standard well-depth procedure.

There is another remark about not consistency of the W-S potential with the nuclear density distribution. In the self-consistent calculations it is shown [33] that the single particle potential should have no symmetry in the surface thickness to be consistent with the nuclear density. The internal part is to be considerable larger  $t_{0.5} - t_{0.9} > t_{0.1} - t_{0.5}$  where  $t_{0.5}$  means the half-potential radius.

If for the light nuclei with  $Z \leq 14$  the W-S potential is good, one has an opportunity to merge two approaches in case of heavier nuclei. In the framework of the  $\alpha$ -cluster model a nucleus is considered as a core (an  $\alpha$ -cluster liquid drop, which grows with  $A$ ) and a molecule [26, 27]. The last nucleon is supposed to be in the mean field of the molecule. So, with growing  $A$  the last nucleon potential stays unchanged, but the Coulomb potential for the last proton grows. In FIG. 3 a sketch of the spherical symmetrical potential is presented. In such a representation the nuclear field of the nucleus comes from addition of the potentials of the core and the molecule, so the requirement of larger internal part of the nucleus potential is fulfilled here. The parameters of the molecule potential can be found under the EPN condition for the nucleus corresponding to the molecule. The center of mass of the molecule is shifted towards the periphery on the value  $\Delta$ , which grows with  $A$ . At  $0 \leq r \leq \Delta$  the single particle wave function  $\psi(r) = 0$  and at  $r \geq \Delta$  the wave function  $\psi(r) = C\psi'(r')$  where  $\psi'(r')$  is the wave function of the last nucleon in cms of the molecule placed on the periphery of the Coulomb potential of a

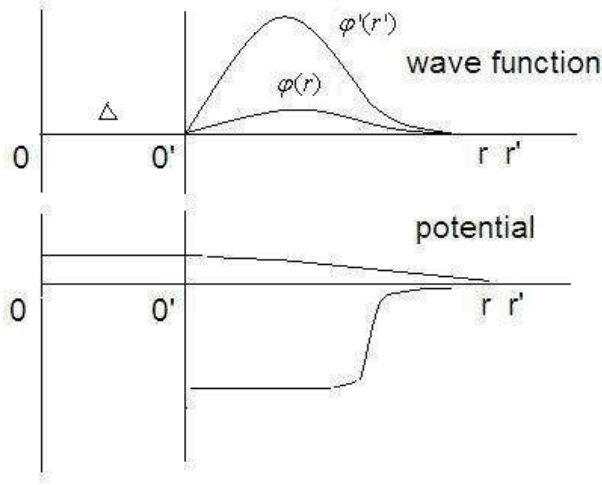


FIG. 3: Local W-S potential with the Coulomb potential for the last proton in a nucleus with  $Z \geq 15$  and the proton's wave function

nucleus  $A$  and  $r' = r - \Delta$ . The  $C$  is determined by the normalization of the functions  $\int_0^\infty \psi'(r')^2 r'^2 dr' = 1$  and  $\int_\Delta^\infty \psi(r)^2 r^2 dr = 1$ . It will bring some other values of ANC and SF for heavier nuclei. Such representation will remove the mentioned above discrepancy and will provide a proper value  $R_0$ . Besides, the function will help in solving the long standing problem of selecting optical potential parameters used for description of input and output elastic channels of the reaction, because the internal part of the amplitude of the reaction will be naturally cut off at small radii, which will make differences of the values of the parameters of different optical potential sets less important for the calculated cross sections. Technically the potential can be used in numerical solutions of the Schrödinger equation without other difficulty. The parameter  $\Delta$  can be estimated in the framework of the  $\alpha$ -cluster model or it can be found in the well-depth procedure at the other parameters fixed to adjust the experimental value of the single particle binding energy. The shell effects in the binding energy, which are restricted within a few MeV, may come from varying the number of the  $\alpha$ -clusters of the molecule from two to five. Practical application of this idea is a subject of another article.

A simple phenomenological proof of this representation can be found in the values of the experimental binding energy of the last neutron in the symmetrical nuclei heavier than the nucleus  $^{12}\text{C}$ . They group around two values  $\varepsilon_n = 15$  MeV for even  $Z$  and  $\varepsilon_n = 11$  MeV for odd  $Z$  (see Table II). So it means that the last neutron has two kinds of links in dependence on whether there is a single  $np$ -pair or not. The energy of neutron separation

in odd( $Z$ )-odd ( $A$ ) stable nuclei (according to the model these nuclei consist of a core made of  $\alpha$ -clusters,  $nn$ -pairs placed in the core, and the molecule of  $^{15}\text{N}$ ) is also within few MeV around the value of the neutron separation energy in the nucleus  $^{15}\text{N}$   $\varepsilon_n = 11$  MeV. For example, for the nuclei  $^{19}\text{F}$ ,  $^{27}\text{Al}$ ,  $^{35}\text{Cl}$ ,  $^{89}\text{Y}$ ,  $^{141}\text{Pr}$  and  $^{209}\text{Bi}$   $\varepsilon_n = 10, 13, 13, 12$  and  $9$  MeV which also says about similar conditions of binding of the last neutron determined by the links with the single  $np$ -pair and three  $\alpha$ -clusters in its close vicinity.

## V. CONCLUSION

For some symmetrical nuclei  $A = B + n/p$  with  $5 \leq Z \leq 14$  the W-S potential parameters satisfying the EPN condition have been found. The Coulomb radius, which is the crucial parameter determining the other parameters in such calculations, is obtained by using the last proton position radius  $R_p$  in the cms of a nucleus  $A$ . The values  $R_p$  are estimated from analysis of binding energies in the framework of the  $\alpha$ -cluster model. The rms radii for the last proton  $\langle r_p^2 \rangle^{1/2}$  in case of the nuclei with  $Z \leq 14$  are in agreement with  $R_p$ . In cases of pure peripheral reactions with one nucleon transfer reactions like  $A(d, t)B$  or  $B(^3\text{He}, d)A$  the error of spectroscopic factors obtained by means of the DWBA analysis with standard potential parameters is estimated. It is shown that the error for some nuclei is larger than 20%.

Analysis shows that the W-S single particle bound state potential satisfying the EPN condition in case of the nuclei with  $Z \geq 15$  produces rms radius for the last proton bigger than  $R_p$ . The radius of the potential increases with a leap and the potential radii are obviously too big to be consistent with the experimental radii. The standard parameters for these nuclei provide better fitting of rms radii to  $R_p$  than the parameters of the EPN condition. But the potential radii are not consistent with the experimental charge radii. The better result is provided by domination of the proton potential over neutron one, which does not seem reasonable. All this allows one to conclude that for the nuclei with  $Z \geq 15$  the W-S potential is inappropriate to represent single particle bound state potential.

According to the  $\alpha$ -cluster model the potential well of the last nucleon is to be of a smaller radius due to the short range of nuclear force and it should be shifted to the nuclear periphery. An idea is proposed that for the DWBA analysis of one nucleon transfer reactions in case of the nuclei with  $Z \geq 15$  the nuclear potential well for the last nucleon should be represented as a local W-S potential covering the periphery of the nucleus.

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